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CERTIFICATION COURSE

Lecture 12: Plasmons- III

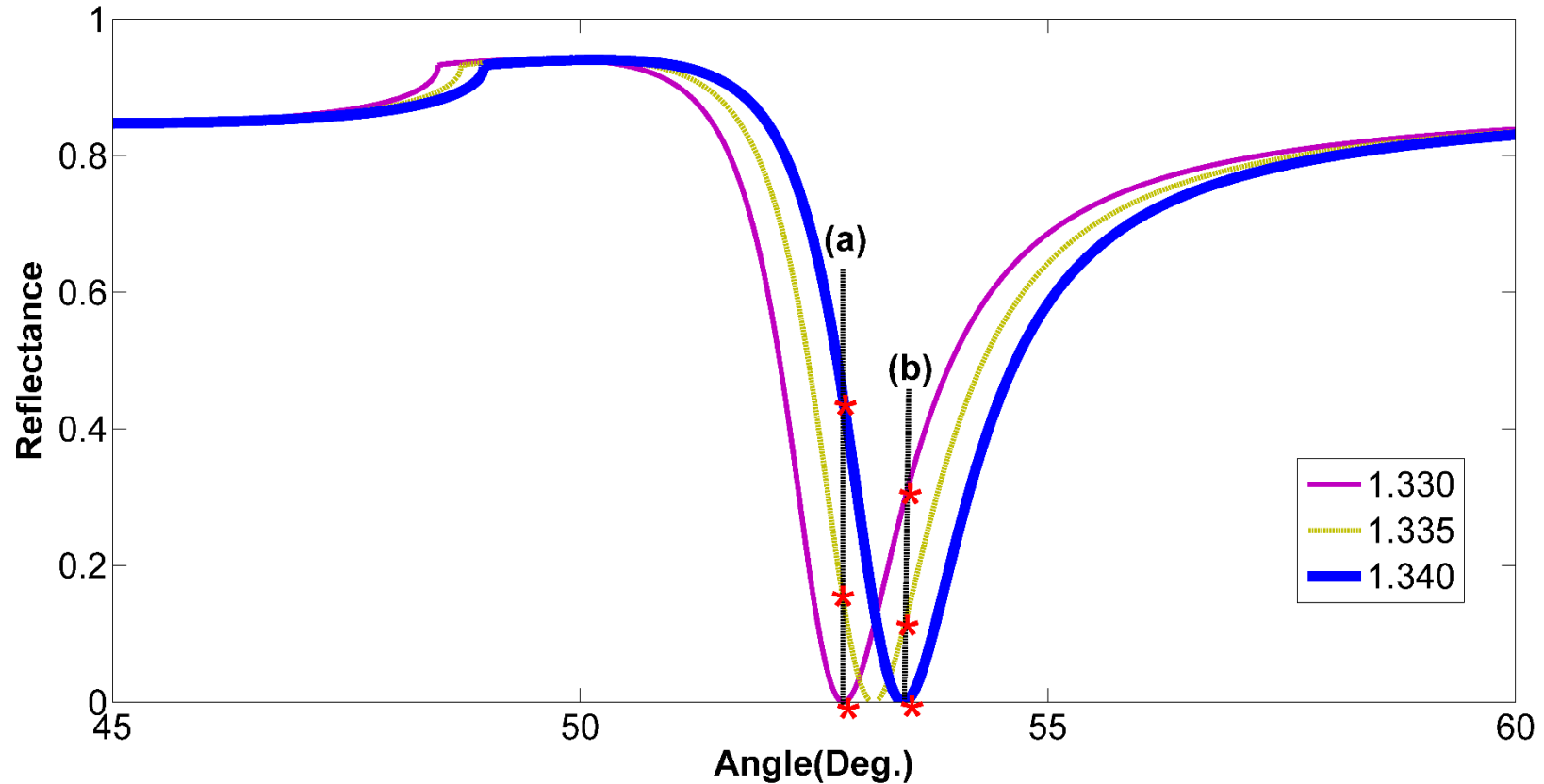
Surface Plasmon: Intensity & Phase Interrogation, Simulations methods, Optimization

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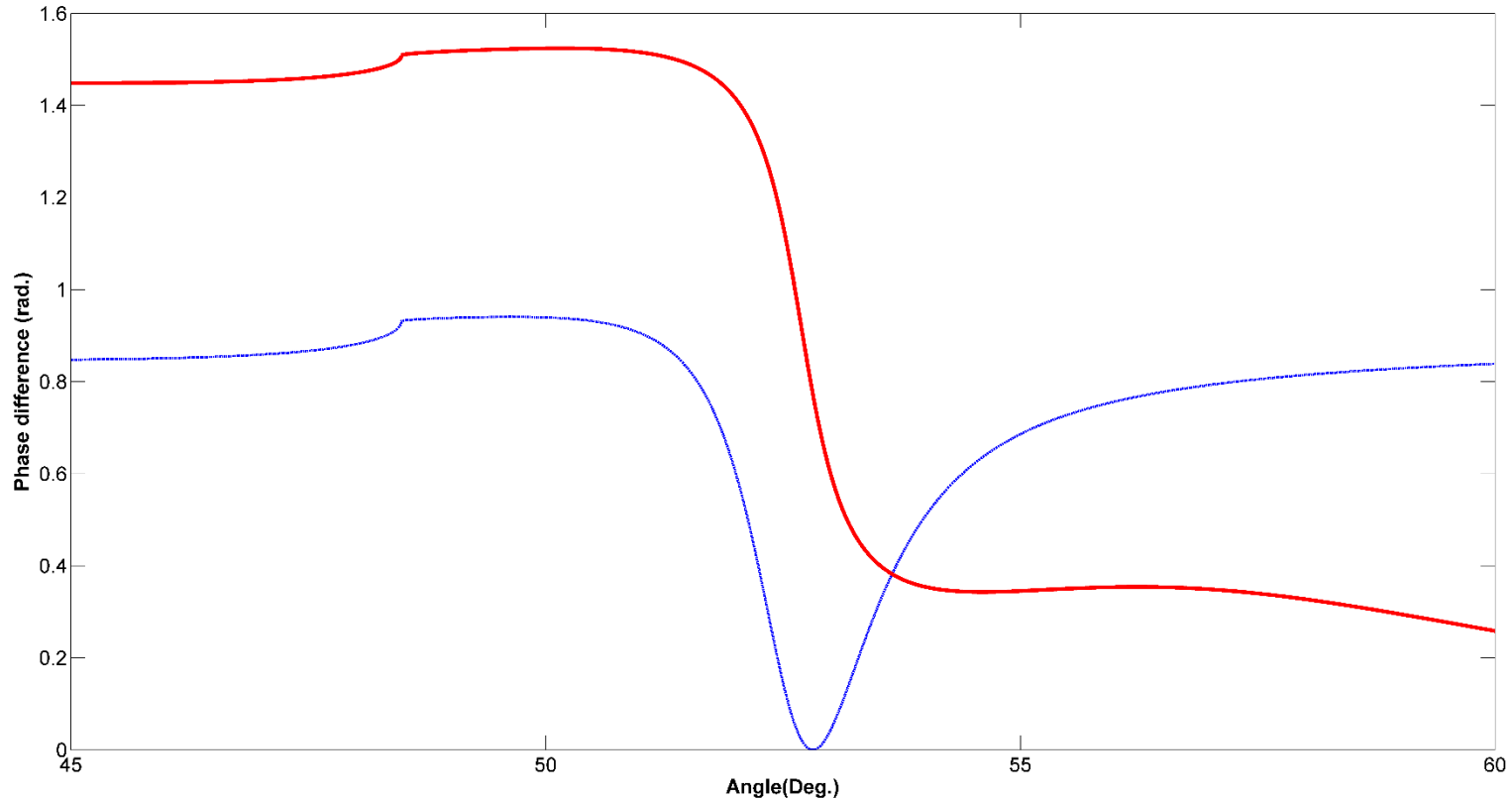
Department of Physics



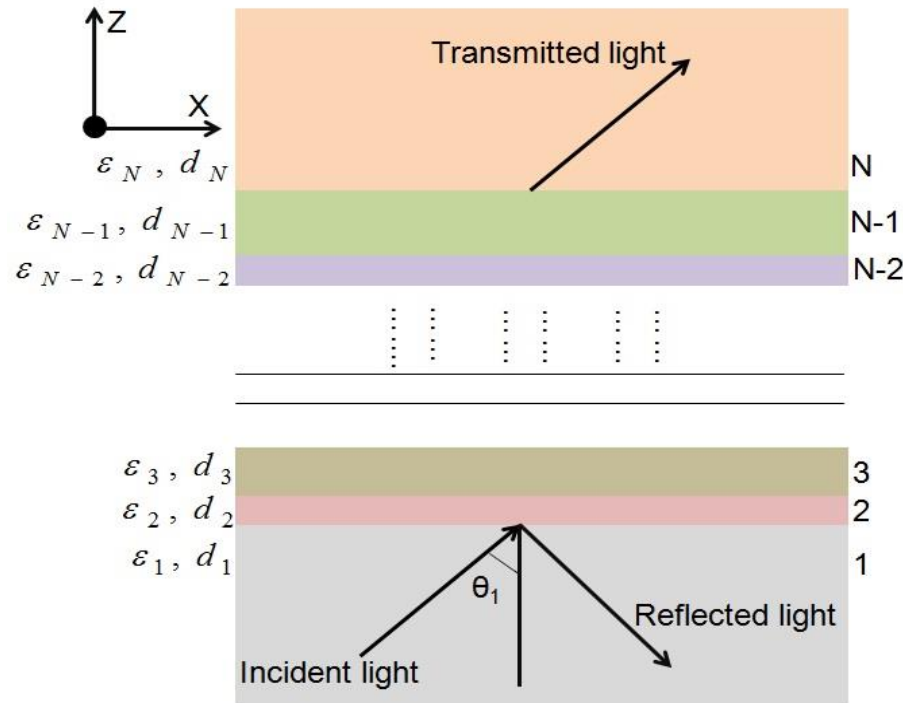
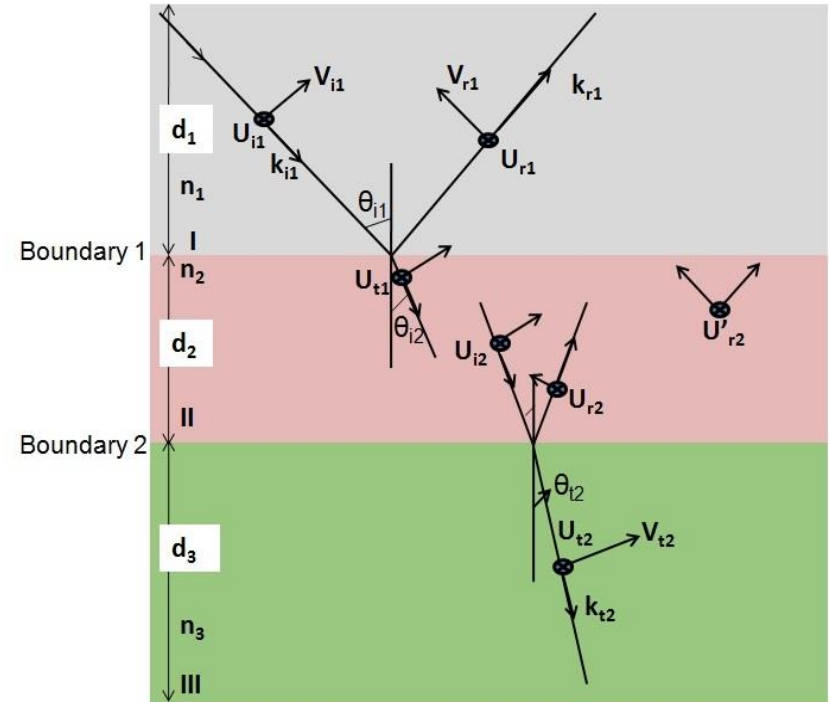
Intensity Interrogation



Phase Interrogation

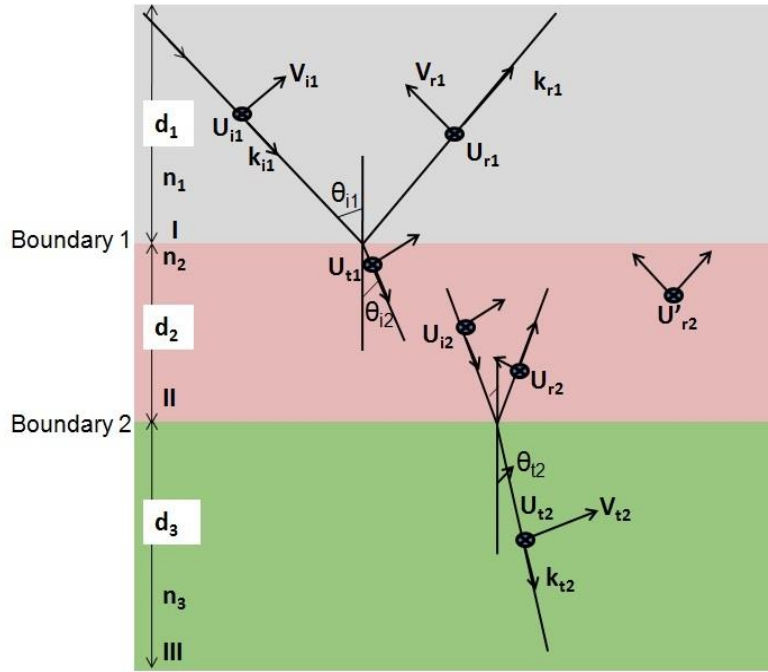


N-Layer Model (Simulating a multilayer structure)



Relation between U & V

$$V = \sqrt{\frac{\epsilon_1}{\mu_1}} n_1 \hat{k} \times U$$



Apply boundary conditions for Electric (U) and magnetic ($V=B/\mu$) field

Interface 1

$$U_1 = U_{i1} + U_{r1} = U_{t1} + U'_{r2}$$

$$V_1 = \sqrt{\frac{\epsilon_1}{\mu_1}} (U_{i1} - U_{r1}) n_1 \cos \theta_{i1} = \sqrt{\frac{\epsilon_1}{\mu_1}} (U_{t1} - U'_{r2}) n_2 \cos \theta_{i2}$$

Interface 2

$$U_2 = U_{i2} + U_{r2} = U_{t2}$$

$$V_2 = \sqrt{\frac{\epsilon_1}{\mu_1}} (U_{i2} - U_{r2}) n_2 \cos \theta_{i2} = \sqrt{\frac{\epsilon_1}{\mu_1}} U_{t2} n_3 \cos \theta_{t2}$$

Phase Shift in traversing a film of thickness d_2

$$\beta_2 = \frac{2\pi d_2}{\lambda} (\varepsilon_2 - n_1^2 \sin^2 \theta_1)^{1/2}$$

Field equations at boundary 2

$$U_{i2} = U_{t1} e^{-i\beta_2} \qquad U_{r2} = U'_{r2} e^{i\beta_2}$$

$$\Rightarrow U_2 = U_{t1} e^{-i\beta_2} + U'_{r2} e^{i\beta_2}$$

$$V_2 = \sqrt{\frac{\varepsilon_1}{\mu_1}} (U_{t1} e^{-i\beta_2} - U'_{r2} e^{i\beta_2}) n_2 \cos \theta_{i2}$$



After simplification, we get

$$U_1 = U_2 \cos \beta_2 - V_2 (i \sin \beta_2) / q_2$$

$$V_1 = U_2 q_2 (-i \sin \beta_2) + V_2 (\cos \beta_2)$$

where

$$q_2 = \frac{(\epsilon_2 - n_1^2 \sin^2 \theta_{i1})^{1/2}}{\epsilon_2}$$

In matrix form

$$\begin{bmatrix} U_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} \cos \beta_2 & (-i \sin \beta_2) / q_2 \\ -i q_2 \sin \beta_2 & \cos \beta_2 \end{bmatrix} \begin{bmatrix} U_2 \\ V_2 \end{bmatrix}$$

or

$$\begin{bmatrix} U_1 \\ V_1 \end{bmatrix} = m_2 \begin{bmatrix} U_2 \\ V_2 \end{bmatrix}$$

Electric and magnetic fields at the first boundary $Z = Z_1$ are related to those at the final boundary $Z = Z_{N-1}$ by

$$\begin{bmatrix} U_1 \\ V_1 \end{bmatrix} = M \begin{bmatrix} U_{N-1} \\ V_{N-1} \end{bmatrix}$$

U_{N-1} and V_{N-1} are the corresponding fields at the boundary of $N-1^{th}$ and N^{th} layers

$$M = \prod_{k=2}^{N-1} m_k = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

- k is the number of layers amongst 1^{st} and N^{th} layer
- There are $N-1$ interfaces

Generalized Matrix for any (k^{th}) layer

$$m_k = \begin{bmatrix} \cos \beta_k & (-i \sin \beta_k) / q_k \\ -i q_k \sin \beta_k & \cos \beta_k \end{bmatrix}$$

Where

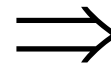
$$q_k = \left(\frac{\mu_k}{\varepsilon_k} \right)^{1/2} \cos \theta_k = \frac{(\varepsilon_k - n_1^2 \sin^2 \theta_1)^{1/2}}{\varepsilon_k}$$

and

$$\beta_k = \frac{2\pi}{\lambda} n_k \cos \theta_k (z_k - z_{k-1}) = \frac{2\pi d_k}{\lambda} (\varepsilon_k - n_1^2 \sin^2 \theta_1)^{1/2}$$

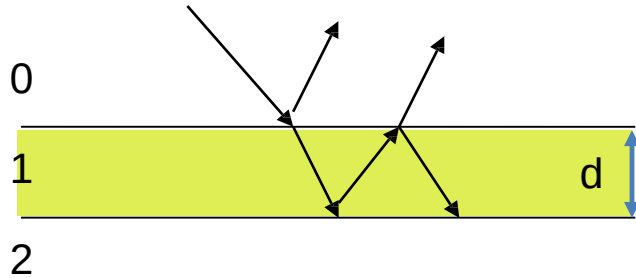
Amplitude Reflection Coefficient for 3-layer structure

$$r_p = \frac{(M_{11} + M_{12}q_3)q_1 - (M_{21} + M_{22}q_3)}{(M_{11} + M_{12}q_3)q_1 + (M_{21} + M_{22}q_3)}$$



$$R_p = |r_p|^2$$

Thin Film Thickness Effect



Airy formula

$$R = \left| r_{012}^p \right|^2 = \left| \frac{E_r^p}{E_0^p} \right|^2 = \left| \frac{r_{01}^p + r_{12}^p \exp(2jk_{z1}d)}{1 + r_{01}^p r_{12}^p \exp(2jk_{z1}d)} \right|^2$$

Where

$$r_p = \left(\frac{k_{zi}}{\varepsilon_i} - \frac{k_{zj}}{\varepsilon_j} \right) / \left(\frac{k_{zi}}{\varepsilon_i} + \frac{k_{zj}}{\varepsilon_j} \right)$$

The dispersion relation
(waveguide condition):

$$1 + r_{01}^p r_{12}^p \exp(2jk_{z1}d) = 0$$

Dispersion Relation

The poles of the reflectivity function give the waveguide modes:

$$1 + r_{01}^p r_{12}^p \exp(2jk_{z1}d) = 0$$

For the symmetric case ($\epsilon_0 = \epsilon_2$) it reduces to:

$$\epsilon_1 k_{z2} + \epsilon_2 k_{z1} \tanh(k_{z1}d / 2i) = 0$$

$$\epsilon_1 k_{z2} + \epsilon_2 k_{z1} \coth(k_{z1}d / 2i) = 0$$

For thick metal film (large d) we get: $\epsilon_1 k_{z2} + \epsilon_2 k_{z1} = 0$

Which together with:

$$k_{zi} = \sqrt{\left(\frac{\omega}{c}\right)^2 \epsilon_i - k_x^2} \Rightarrow k_x = \frac{\omega}{c} \sqrt{\frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2}}$$

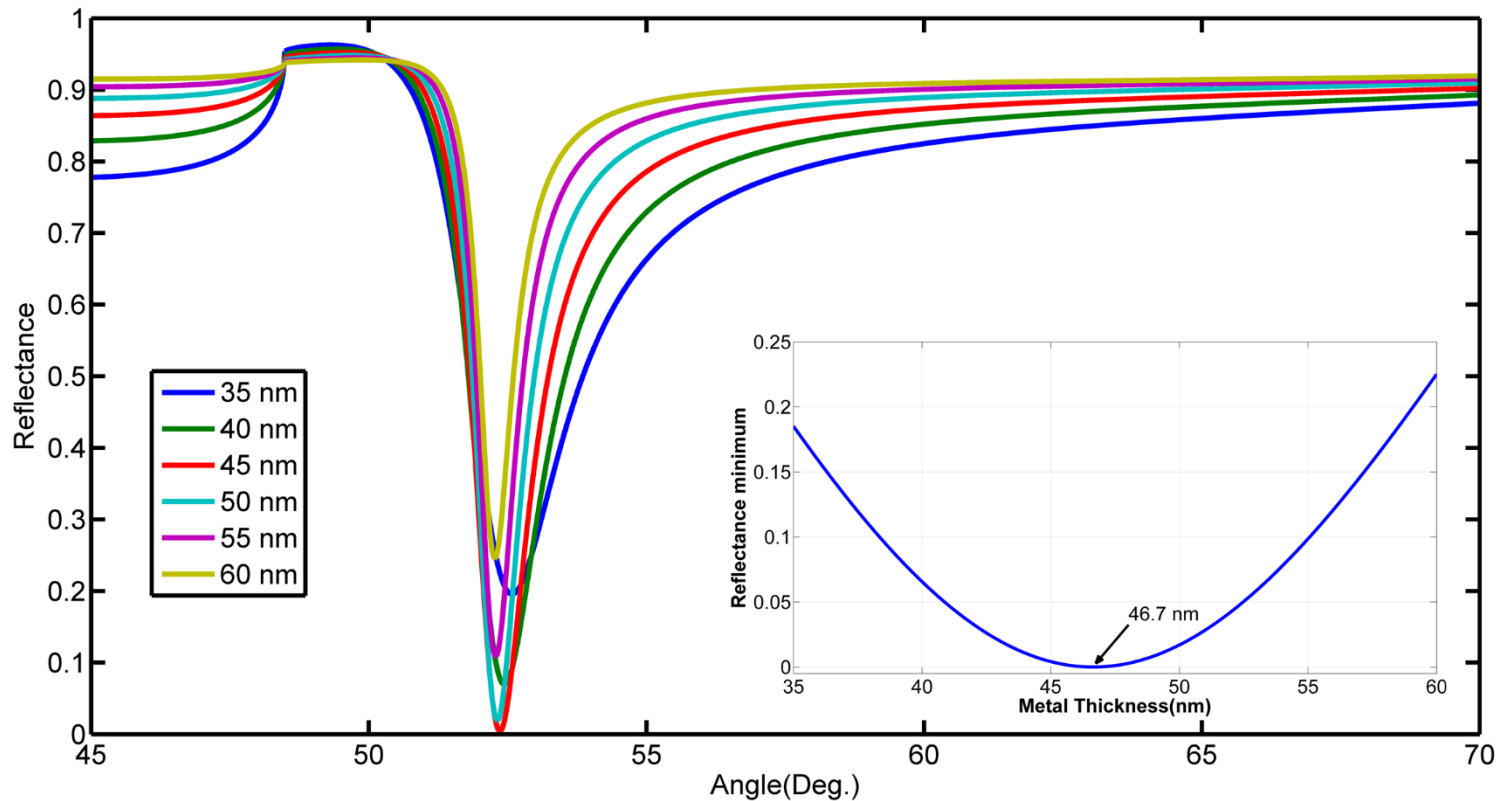
SPR Shift due to Thin Metal Film

When the metal film becomes thin, the dielectric above (glass prism for example) starts to affect the resonance and causes a shift of the SPR wave-vector by:

$$\text{Re}\{\Delta k_x\} = \text{Re}\left\{2r_{01}^p \exp(2jk_{z1}d) \left(\frac{\omega}{c}\right) \left(\frac{\epsilon_1\epsilon_2}{(\epsilon_1 + \epsilon_2)}\right)^{3/2} \left(\frac{1}{(\epsilon_1 - \epsilon_2)}\right)\right\}$$

Depending on the thickness, the light reflected and the light released by the plasmons interfere destructively. This will happen at an optimal thickness d at which they will be totally out of phase. The typical thickness is approximately 50nm.

Optimum Plasmonic Film Thickness



Summary

1. Intensity and Phase interrogation techniques were discussed.
2. The N-layer model was studied to simulate multilayer optical structures.
3. The optimization of the thickness of thin metal film in Kretschmann configuration was discussed.

Thank You

